

Mathematics Assessment Project  
Formative Assessment Lesson Materials

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# Representing Data Using Frequency Graphs

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MARS Shell Center  
University of Nottingham & UC Berkeley  
Alpha Version

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# Representing Data Using Frequency Graphs

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## Mathematical goals

This lesson unit is intended to help you assess how well students are able to:

- Interpret data on a single count or measurement variable using frequency graphs and box plots.
- Understand how a large number of data points may result in the graph being approximated by a continuous distribution.

## Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

**S-ID** Summarize, represent, and interpret data on a single count or measurement variable.

This lesson involves a range of *mathematical practices*, with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

## Introduction

The unit is structured in the following way:

- Before the lesson, students complete an assessment task individually that is designed to reveal their current understanding.
- A whole class introduction provides students with guidance on how to work through the task. Students work in pairs or threes on a collaborative discussion task, matching written interpretations and graphs as they begin to link these two representations.
- After a whole-class plenary discussion, students revise their solutions to the assessment task.

This lesson can be taught in conjunction with the lesson *Representing Data Using Box Plots* or independently.

## Materials required

Each student will need a copy of the Assessment task: *Cell Phones 1*, a mini-whiteboard, a pen and an eraser.

Each small group of students will need the following resources:

- Card Set: *Frequency Graphs*, and Card Set: *Interpretations*. Cut-up both card sets before the lesson.
- One large sheet of paper and a glue stick for making posters.

There are also some projector resources to help with whole class discussions.

## Time needed

Approximately 15 minutes before the lesson, a one hour lesson and a follow-up homework. Exact timings will depend on the needs of the class.

## Before the lesson

### Assessment task: *Cell Phones* (15 minutes)

Give this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task *Cell Phones 1*.

*Read through the questions and try to answer them as carefully as you can.*

It is important that students are allowed to answer the questions without your assistance, as far as possible.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

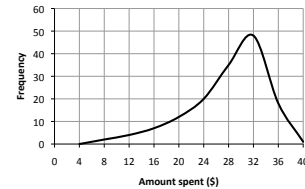
### Assessing students' responses

Collect students' responses to the task, and note down what their work reveals about their current levels of understanding and their different approaches.

We suggest that you do not grade students' work. The research shows that this will be counterproductive, as it will encourage students to compare their grades and distract their attention from the mathematics. Instead, to help students make further progress, prepare some questions that will enable students to re-engage with their work. Some suggestions for appropriate questions are given on the next page. At this stage, either write a list applicable to your own class, using a selection of these and your own questions, or if you have enough time, add appropriate questions to each piece of student work. Students will use these questions at the end of the next lesson to help improve their answers.

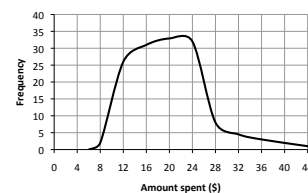
**Cell Phones 1**

Here is a frequency graph that shows the monthly spending of a group of students on their cell phones:



The graph shows the students spent between \$5 and \$40 a month, a range of \$35. The modal amount spent is about \$31. The median is about \$29. Most students spent over \$25 each month.

Here is another frequency graph that shows the monthly spending of a second group of students on their cell phones:



Describe what this graph shows:

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Common issues	Suggested questions and prompts
<b>Limited or incorrect description of the graph</b> For example: Student mentions just the lowest and highest scores.	<ul style="list-style-type: none"> <li>• <i>What can you tell me about other mathematical measures?</i></li> <li>• <i>What can you tell me about the monthly spending of most of the students?</i></li> </ul>
<b>Student assumes the median value is equal to the mode or student assumes the median value is exactly half way between the maximum and minimum value</b>	<ul style="list-style-type: none"> <li>• <i>What proportion of students spends less than the median value? How can you show this on your graph? [The value at the point where the area under the curve is split in half.]</i></li> <li>• <i>What does the median represent?</i></li> </ul>
<b>Student does not contextualize the data</b> For example: The student states the mode is 23, maximum and minimum values are 5 and 44 respectively, but does not refer to the context.	<ul style="list-style-type: none"> <li>• <i>What do these figures represent?</i></li> </ul>

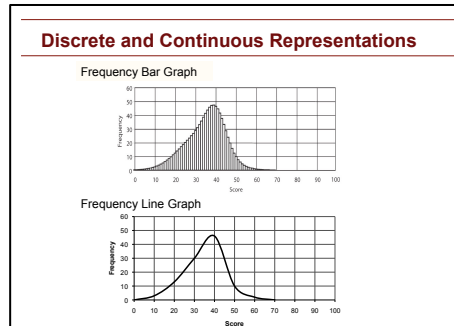
## Suggested lesson outline

Throughout this lesson, encourage students to use the correct mathematical language.

### Whole class introduction: Interpreting Graphs (10 minutes)

Give each student a mini-whiteboard, an eraser and a pen.

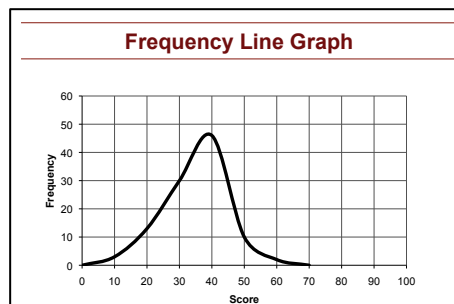
Show Slide 1 of the projector resource:



Explain to students that the bar chart represents the scores of students in a test for which the maximum score was 100.

When there are many bars close together, the data can be represented as a continuous line, that is a frequency graph. This makes it a little easier to read off the values.

Now show Slide 2 of the projector resource:



Students are to write on their mini-whiteboards all the information they can derive from the graph. After a couple of minutes ask students to show you their answers.

Ask one or two students to justify their answers. Even if their explanations are incorrect or only partially correct, write them next to the graph. Encourage students to challenge these interpretations and then replace them with new ones.

You may want to ask students a selection of the following questions in turn:

*What can you say about the test? Did the students find it difficult or easy? How can you tell?*

*What is the range of scores?*

*What is the range of scores for most students?*

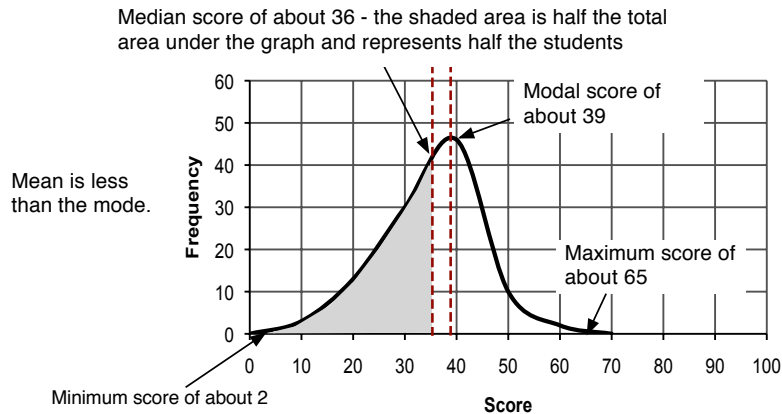
*What is the mode score?*

*Do you think the median score is higher or lower than the mode score? How can you tell?*

*Do you think the mean score is higher or lower than the mode? How can you tell?*

*Roughly how many candidates took the test? [About 1,000.] How do you know?*

The graph may end up looking like this:



This introduction models how students should work during the collaborative task.

### Collaborative activity: Matching Card Sets A and B (20 minutes)

Ask students to work in small groups of two or three students.

Give each group the Card Set: *Frequency Graphs*, Card Set: *Interpretations*, a large sheet of paper for making a poster, and a glue stick.

*Take turns at matching pairs of cards that you think belong together.*

*Each time you do this, explain your thinking clearly and carefully. Your partner should either explain that reasoning again in his or her own words, or challenge the reasons you gave.*

*You both need to be able to agree on and explain the placement of every card.*

*Write your reasons for the match on the cards or the poster.*

*Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them.*

These instructions are summarized on Slide 3 of the projector resource.

While students work in small groups you have two tasks, note different student approaches to the task, and support student reasoning.

### Note different student approaches to the task

In particular, notice any difficulties that students encounter with what they are doing, and the ways they justify and explain to each other. Do students check to see if their match is correct? Do they assume the scores go up the vertical axis? How do they understand how to use the graph to find the median and mode? Are students using the correct mathematical language? Are students using all the information on the cards or just the first sentence? What do they do if they get stuck?

You can then use this information to focus your questioning in the whole class discussion towards the end of the lesson.

### Support student reasoning

Try not to make suggestions that move students towards particular matches. Instead, ask questions to help students to reason together. You may want to use some of the questions and prompts from the *Common Issues* table.

If a student struggles to get started, encourage them to ask a specific question about the task. Articulating the problem in this way can sometimes offer a direction to pursue that was previously overlooked. However, if the student needs their question answered, ask another member of the group for a response.

Here are some further questions you may want to use:

*Rewrite the description in your own words.*

*Write a description of the graph.*

*Tell me how you have used **all** the information [i.e. both sentences] on the Interpretation card to match it with a graph.*

*How can you figure out an approximate value for the median?*

*Did many students get a low/high score for this graph? How do you know?*

*Show me a graph that shows the median score equal to the mode score? How do you know?*

*Show me a graph in which a lot of students/few students found it easy? How do you know?*

*Which graph shows students of a similar ability? How do you know?*

*Make up five figures where the median is greater than the mode. Now sketch a graph of these figures.*

If you find one student has matched two cards, challenge another student in the group to provide an explanation.

*Danny matched these cards. Ezra, why does Danny think these two cards go together?*

If you find the student is unable to answer this question, ask them to discuss the work further. Explain that you will return in a few minutes to ask a similar question.

If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold a whole class discussion.

### **Sharing posters (10 minutes)**

Give each small group a glue stick.

As students finish matching the cards, ask one student from each group to visit another group's poster.

*If you are staying at your desk, be ready to explain the reasons for your group's matches.*

*If you are visiting another group, copy your matches onto a piece of paper.*

*Go to another group's desk and check to see which matches are different from your own.*

*If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.*

*When you return to your own desk, you need to consider as a group whether to make any changes to your poster.*

You may want to use Slide 4 of the projector resource to display these instructions.

When students are satisfied, they are to glue the equations onto the poster

**Plenary discussion (15 minutes)**

The intention is that this discussion focuses on the justification of one or two examples that students found difficult, rather than checking students all have all solutions correct. In trials students have had difficulty matching graphs B, C, and H.

*How did you decide to match this card?*

*Can someone else put that into their own words?*

*Could this card be matched with another one?*

*Which graph card do you think is unrealistic? Why?*

To encourage good explanations, you could ask students for examples of matches that they have explained well.

Now ask students to sketch on their whiteboard a graph that shows the test results of a different class. The first piece of information about the test is that it resulted in a huge range of scores.

Once students have shown you their whiteboards, ask them to swap whiteboards with a neighbor. They are to write a second piece of information about the test on their neighbors' whiteboard. This piece of information combined with the first should make their existing graph incorrect.

Once whiteboards are returned, students will now need to re-draw their graph so that it represents both pieces of information.

Ask students to show you their whiteboards. Ask a few students with differing graphs to explain why they were forced to re-draw it.

**Improving individual solutions to the assessment task (10 minutes)**

Return to the students their original assessment *Cell Phones 1*.

*Look at your original responses and think about what you have learned from this lesson.*

*Using what you have learned, try to improve your work.*

*When you revise your work, write as if you are explaining the solutions to someone unfamiliar with this type of math.*

Ask students to use a different color pen to the one they originally used.

If you have not added questions to individual pieces of work, then write your list of questions on the board.

Students should select from this list only the questions they think are appropriate to their own work. If students may have difficulty selecting questions then they are to answer all of them. Doing this should help them recognize whether the question does apply to their work.



**Solutions**

**Assessment task: Cell phones 1**

1. Here is a possible description of the graph; however, the measures are all approximate:

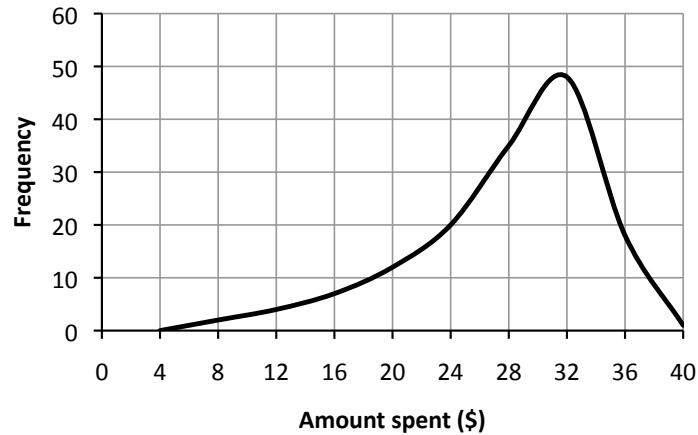
The graph shows the students spent between \$6 and \$44 a month, a range of \$38. The mode amount spent is about \$23. The median is about \$18. The majority of students spend between \$23 and \$24 a month. Very few students spend more than \$30 a month.

**Collaborative activity:**

Frequency Graph	Interpretation	Frequency Graph	Interpretation
A	8	E	3
B	1	F	5
C	2	G	6
D	7	H	4

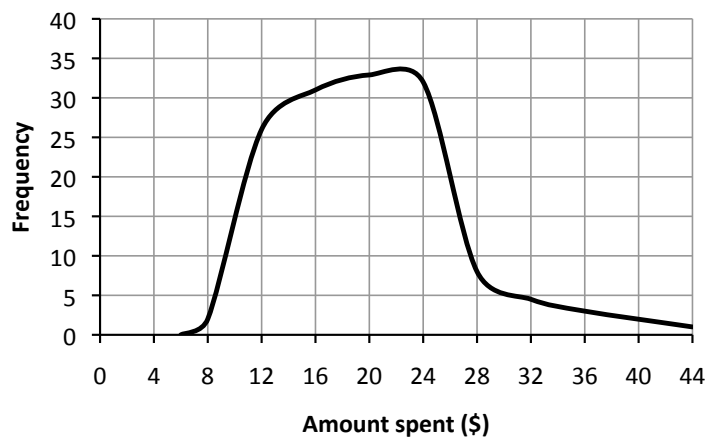
# Cell Phones 1

Here is a frequency graph that shows the monthly spending of a group of students on their cell phones:



**The graph shows** the students spent between \$5 and \$40 a month, a range of \$35. The modal amount spent is about \$31. The median is about \$29. Most students spent over \$25 each month.

Here is another frequency graph that shows the monthly spending of a second group of students on their cell phones:



Describe what this graph shows:

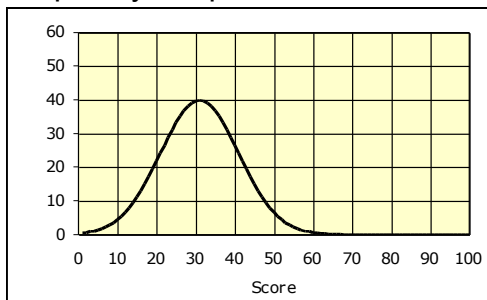
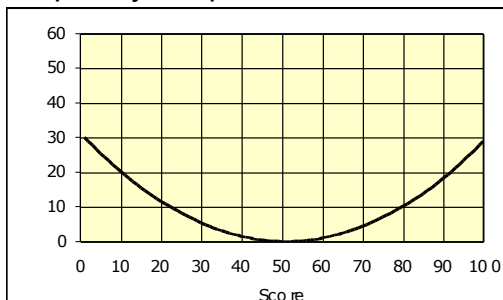
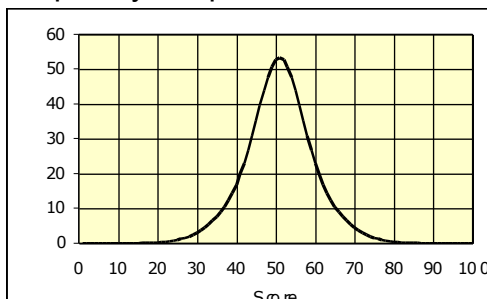
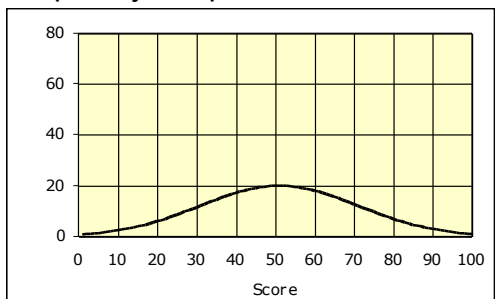
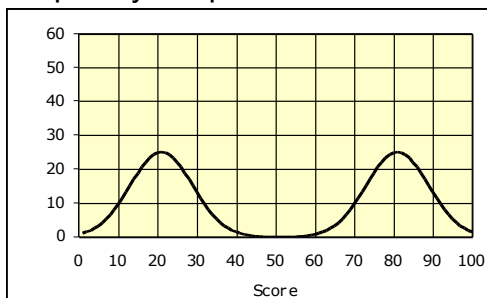
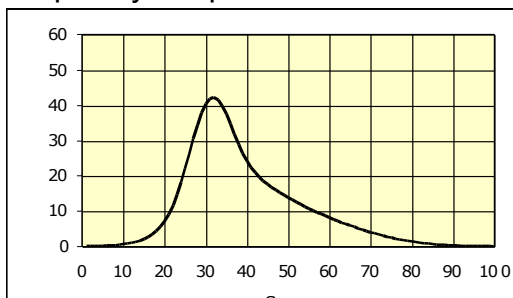
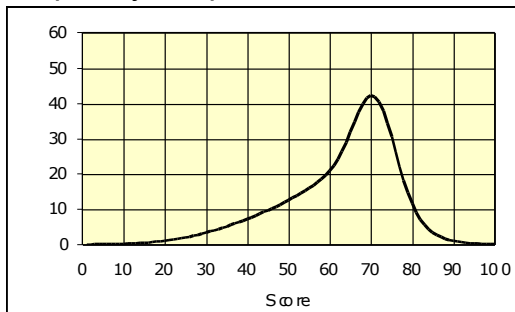
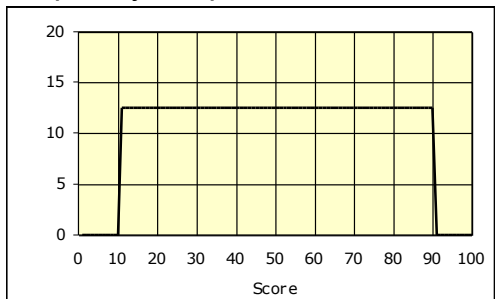
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**Card Set: Frequency Graphs****Frequency Graph A****Frequency Graph B****Frequency Graph C****Frequency Graph D****Frequency Graph E****Frequency Graph F****Frequency Graph G****Frequency Graph H**

**Card Set: Interpretations**

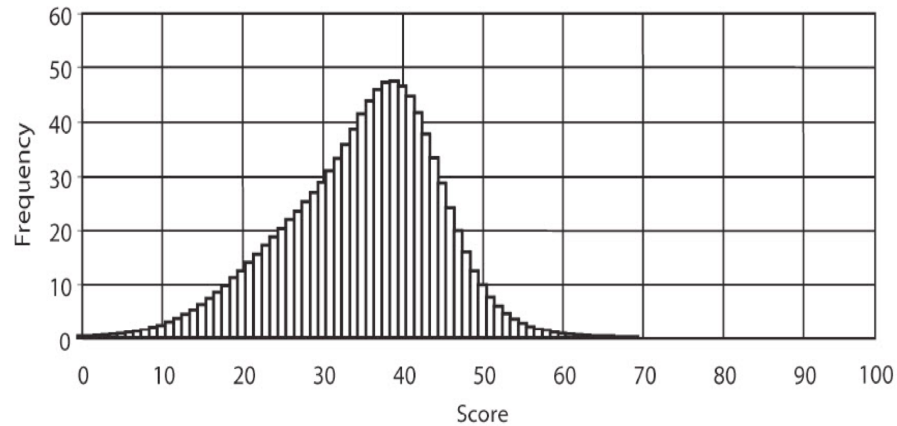
1.  This was the sort of test where you could either do everything or you couldn't get started.	2.  This test did not sort out the stronger students from the weaker ones. They all got similar scores.
3.  Two groups of students sat the test. One group had studied the work for two years. The other group had only just begun.	4.  This test resulted in a huge range of scores. Everyone could do something but nobody could do it all.
5.  In this test, the mean score was greater than the modal score.	6.  In this test, the mean score was smaller than the modal score.
7.  In this test, the median and the modal scores were the same. There was a very big range of scores.	8.  This test was much too difficult for most people.

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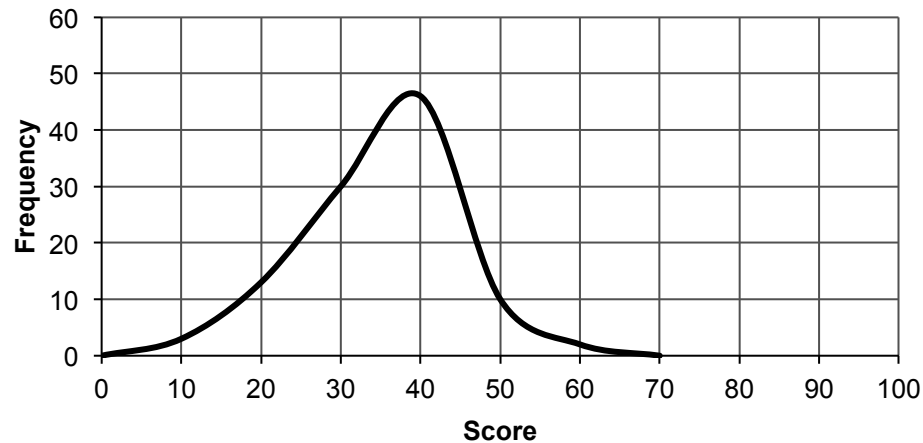
# Discrete and Continuous Representations

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Frequency Bar Graph



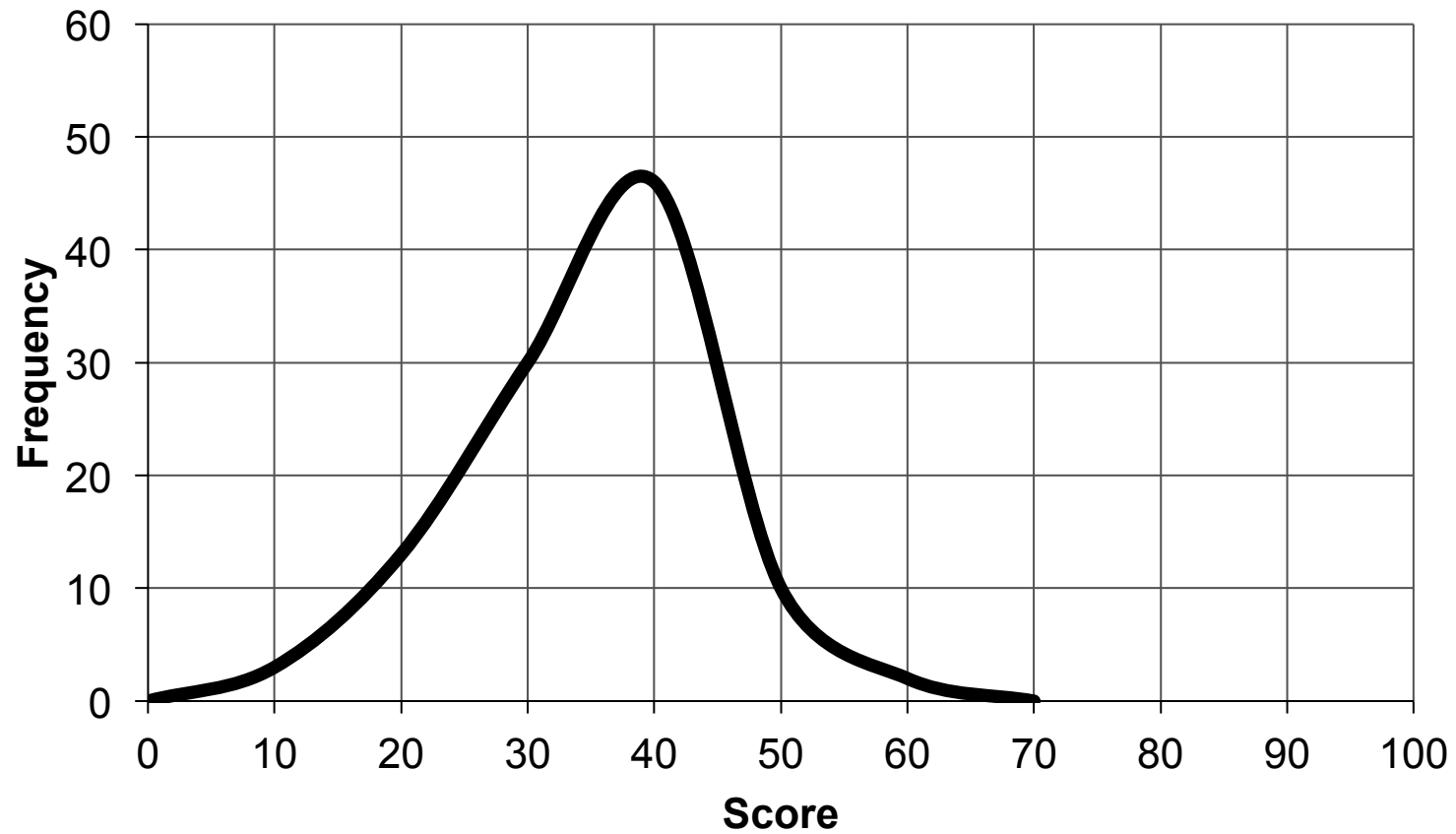
Frequency Line Graph



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# Frequency Line Graph

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# Matching Cards

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1. Take turns at matching pairs of cards that you think belong together.
2. Each time you do this, explain your thinking clearly and carefully.
3. Your partner should either explain that reasoning again in his or her own words, or challenge the reasons you gave.
4. You both need to be able to agree on and explain the placement of every card.
5. Write your reasons for the match on the cards or the poster.

You may find some of 'Word' cards match two graphs. This problem will be resolved as you match more cards.

Be prepared to change your mind about the matches.

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# Sharing Posters

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1. If you are staying at your desk, be ready to explain the reasons for your group's matches.
2. If you are visiting another group:
  - Copy your matches onto a piece of paper.
  - Go to another group's desk and check to see which matches are different from your own.
  - If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.
3. When you return to your own desk, you need to consider as a group whether to make any changes to your poster